Scattering of a particle with spin by atomic chain as null test of T-violating P-even magnetism

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Abstract

T-odd P-even long-range electromagnetic interaction of a particle of spin 1/2 with the nucleus is considered. Though matrix element of the interaction is zero for the particles on mass shell, nevertheless, null test exists for the interaction. The test consists in measuring of the spin-dependent T-odd P-even forward elastic scattering amplitude of a particle of spin 1/2 by atomic chain (axis) in a crystall.

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In connection with the direct observation of time-reversal symmetry violation in the system of $K^0 - \bar{K}^0$ mesons [1] it would be interesting to detect T-non-invariance in other nuclear or atomic systems. It is necessary to distinguish P- T- odd interaction from P-even T-odd one. While on the strength constants of the first type interactions there are rather rigid restrictions, obtained from dipole moment of atoms and particles measuring, restrictions on the constants of P-even T-odd interactions are not so strong. As it is known, zero test of the last kind interactions is observation of $\sim (\boldsymbol{\sigma} \times \boldsymbol{n} \cdot \boldsymbol{S})(\boldsymbol{n} \cdot \boldsymbol{S})$ term in the forward elastic scattering amplitude of a particle of spin 1/2 by a particle with a spin $S \geq 1$ [2–5], where $\boldsymbol{n}$ is a unit vector in a direction of an incident particle momentum. Relevant experiments were carried out [6] and plan to be performed on a superconducting synchrotron COSY [7].

In this work new null test of T-invariance is presented, which consists in measuring of the T-odd forward elastic scattering amplitude of a particle with spin by an atomic chain (axis) or a plane.

The motion of a particle with spin in matter can be described with the help of refraction index, depending on a spin [8]: $\mathcal{N} = 1 + \dfrac{2\pi\wp}{p^2} f(0)$, where $f(0)$ is forward scattering amplitude of a particle with spin by a scattering centre of a medium, p is a wave number of a particle, $\wp$ is the density of scatterers in the matter. The dependence of refraction index on the orientation of spin leads to the experimentally observable effects: absorption of a particles beam, depending on the orientation of particle spin (dichroism), rotation and oscillation of a particle spin [9,10].

In passage of a particle under small angles to the atomic chains or planes the scattering occurs not on the separate atom, but on the whole atomic chain or plane.

For the angles, less than Lindhard angle, negatively charged particle is captured by an axis or a plane and goes in its vicinity, i.e. the channelling phenomenon arises [11] (positively

charged particle is localised between neighbour atomic planes). Channelling and radiation under channelling were actively studied earlier [11–13].

But we are interesting the case, when a particle goes under angle more than Lindhard one, at which the channelling, begins. However the angle is so small (less than $\frac{R_a}{d}$, R_a is atomic radius, d is interatomic distance in chain), that the particle motion can be described by the averaged axis or plane potential.

In this case particle refraction index is also determined by the formula above, but the amplitude should be read as denoting particle forward elastic scattering amplitude by the whole axis or plane (accounted for unit axis length or unit area of the plane).

Accordingly, a density of scatterers should be understand as two-dimensional density of cross points of axes, with the plane being perpendicular to the axes, or one-dimensional density of cross points of planes by their normal line. Let's designate the direction of an axes or direction of planes normal line by the unit vector $\boldsymbol{a}$ (fig.1) and consider spin structure of the scattering amplitude at zero angle of a particle with spin $\boldsymbol{S}$ by an axis or a plane:

$$F(0) = A + B(\boldsymbol{S}\boldsymbol{n}) + B_1(\boldsymbol{S}\cdot\boldsymbol{a})(\boldsymbol{n}\cdot\boldsymbol{a}) + C_1(\boldsymbol{S}\times\boldsymbol{a}\cdot\boldsymbol{n})(\boldsymbol{a}\cdot\boldsymbol{n}) + A_1(\boldsymbol{S}\cdot\boldsymbol{n})^2 + \\ + A_2(\boldsymbol{S}\cdot\boldsymbol{a})^2 + A_3(\boldsymbol{S}\cdot\boldsymbol{a})(\boldsymbol{S}\cdot\boldsymbol{n})(\boldsymbol{n}\cdot\boldsymbol{a}) + C_2(\boldsymbol{S}\times\boldsymbol{a}\cdot\boldsymbol{n})(\boldsymbol{S}\cdot\boldsymbol{a}) + \ldots, \quad (1)$$

$\boldsymbol{n}$ being the unit vector in a direction of incident particle momentum. For a particle with a spin 1/2 only three first terms should be in the expression (1), the remained terms occur for particles with spins greater than 1/2. The coefficients $A_n, B_n \cdots$ are even functions of $(\boldsymbol{n}\cdot\boldsymbol{a})$. From the view-point of fundamental symmetries chesking T-violating/P-conserving $\sim (\boldsymbol{S}\times\boldsymbol{a}\cdot\boldsymbol{n})(\boldsymbol{a}\cdot\boldsymbol{n})$ and T-/P-violating $\sim (\boldsymbol{S}\times\boldsymbol{a}\cdot\boldsymbol{n})(\boldsymbol{S}\cdot\boldsymbol{a})$ terms are interesting. Experimental observation of these terms gives novel null test of T-invariance. It turns out to be, that the first term can arise from the T-odd/P-even particle interaction

$$V_T(\boldsymbol{r}) = (\boldsymbol{S}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{r})u^T(r)(\boldsymbol{r}\cdot\hat{\boldsymbol{p}}) + (\hat{\boldsymbol{p}}\cdot\boldsymbol{r})u^T(r)(\boldsymbol{r}\cdot\boldsymbol{S}\times\hat{\boldsymbol{p}}) ,$$

and the second term from the T-/P-odd

$$V_{PT}(\boldsymbol{r}) = (\boldsymbol{S}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{r})u^{PT}(r)(\boldsymbol{r}\cdot\boldsymbol{S}) + (\boldsymbol{r}\cdot\boldsymbol{S})u^{PT}(r)(\boldsymbol{r}\cdot\boldsymbol{S}\times\hat{\boldsymbol{p}}) ,$$

one. $\hat{\boldsymbol{p}}$ designates operator of particle momentum and $u^{PT}(r)$, $u^T(r)$ being some radial functions. These interactions do not lead to the observable effects in ordinary elastic scattering: evaluation of T-odd amplitude in the Born approximation gives zero. The same result will be if one calculate the scattering amplitude by chain of atoms in the Born approximation. But taking into account the perturbation of the plane waves by the cylindrical Coulomb potential of an axis in the distorted wave approximation [14] leads to the non zero T-odd amplitude.

Let us consider T-violating interaction arising from one photon exchange. T-non-invariant parity conserving electromagnetic interaction vertex of a particle of spin 1/2 with the electromagnetic field can be written as [15,16]

$$\Gamma^\mu_T = \frac{e\,\mathfrak{e}_T}{m^2}((Pq)\gamma^\mu - (\gamma q)P^\mu) - \frac{e\,\mathfrak{m}_T}{2m^3}(Pq)\sigma^{\mu\nu}q_\nu , \quad (2)$$

where $P = p_3 + p_1$, $q = p_3 - p_1$ (fig.2). Considering the spinless charged nuclei we take ordinary electromagnetic vertex in the form ZeP_μ [17], $k = p_4 + p_2$. After the application

of ordinary diagramm technique, we obtain appropriate matrix element due to one photon exchange:

$$\mathcal{M} = \frac{Ze^2 \mathrm{i}}{m^2}\Big\{\mathfrak{e}_T((Pq)\gamma^{\mu}_{31} - (\gamma_{31}q)P^{\mu}) - \frac{e\,\mathfrak{m}_T}{2m}(Pq)\sigma^{\mu\nu}_{31}q_{\nu}\Big\}D_{\mu\eta}(q)k^{\eta} \quad , \tag{3}$$

where $\gamma^{\mu}_{31} \equiv \bar{u}_3(p_3)\gamma^{\mu}u_1(p_1)$, ...and $D_{\mu\nu}(q) = 4\pi g_{\mu\nu}/q^2$ is a photon propagator. Setting $p_1 = p$, $p_3 = p + q$, $q = (0, \boldsymbol{q})$ and substituting $u(p) = \begin{pmatrix} \sqrt{\varepsilon + m}\,\phi \\ (\varepsilon+m)^{-1/2}\,(\boldsymbol{\sigma p})\,\phi \end{pmatrix}$, ($\phi$ is spin wave function of a particle) into (3) we find the T-odd scattering amplitude of a particle by resting nucleus for the small momentum transferred $\boldsymbol{q}$:

$$f(\boldsymbol{q}) = -\frac{\mathcal{M}}{8\pi M} = -\frac{2Ze^2}{m^2}\left(\frac{\mathfrak{m}_T}{m} + \frac{\mathfrak{e}_T}{\varepsilon + m}\right)\frac{(\boldsymbol{pq})(\boldsymbol{\sigma} \times \boldsymbol{p}\cdot\boldsymbol{q})}{q^2}. \tag{4}$$

In evaluation of the amplitude we consider the particle being on mass shell everywhere except for the term $(\boldsymbol{p}\cdot\boldsymbol{q})$. If a particle is completely on mass shell, $(\boldsymbol{p}\cdot\boldsymbol{q}) = 0$ and the amplitude is zero. Dependence of the amplitude on momentum transferred $\boldsymbol{q}$ is the same as for elastic scattering amplitude of magnetic dipoles. So, it turns out to be that the interaction is long-range one.

Improper conclusion have been made earlier [16] (repeated then in the monograph [18]) about non-existence of long-range (i.e. decreasing as $\frac{1}{r^3}$ with distance or weaker [19]) T-odd potential.

We can consider scattering of a particle in a framework of the Schrodinger equation with the relativistic mass [17]:

$$(\nabla^2 + p^2)\Psi(\boldsymbol{r}) = 2\varepsilon V(\boldsymbol{r})\Psi(\boldsymbol{r}). \tag{5}$$

In the Born approximation the amplitude (4) can be obtained from the T-odd interaction, depending on the energy:

$$V_T(\boldsymbol{r}) = \frac{3Ze^2}{2\varepsilon m^2}\left(\frac{\mathfrak{m}_T}{m} + \frac{\mathfrak{e}_T}{\varepsilon + m}\right)\left((\hat{\boldsymbol{p}}\boldsymbol{r})\frac{1}{r^5}(\boldsymbol{r}\cdot\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}) + (\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{r})\frac{1}{r^5}(\boldsymbol{r}\hat{\boldsymbol{p}})\right) , \tag{6}$$

wich can be presented in the form:

$$V_T(\boldsymbol{r}) = -\frac{e}{2\varepsilon m^2}\left(\frac{\mathfrak{m}_T}{m} + \frac{\mathfrak{e}_T}{\varepsilon + m}\right)\left(\hat{\boldsymbol{p}}\cdot\{\boldsymbol{\nabla}\otimes\boldsymbol{E}(\boldsymbol{r})\}\cdot(\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}) + (\boldsymbol{\sigma}\times\hat{\boldsymbol{p}})\cdot\{\boldsymbol{\nabla}\otimes\boldsymbol{E}(\boldsymbol{r})\}\cdot\hat{\boldsymbol{p}}\right) ,$$

where $\boldsymbol{E}(\boldsymbol{r}) = -\boldsymbol{\nabla}\phi(\boldsymbol{r}) = Ze\frac{\boldsymbol{r}}{r^3}$ is a strength of the electric field created by the nucleus at the point $\boldsymbol{r}$. $\otimes$ means a direct vector product. Till now we did not take into consideration screening of the nucleus field by the electrons. To remedy this we take the nucleus electric potential in the form $\phi(r) = \frac{Ze}{r}\mathrm{Erfc}\left(\sqrt{\frac{3}{2}}\frac{r}{R_a}\right)$, where $\mathrm{Erfc}(x) = \frac{2}{\sqrt{\pi}}\int\limits_x^{\infty} \mathrm{e}^{-t^2}dt$ and R_a is atomic radius. Now, as it is usually done in the theory of channelling, we find the T-odd average potential of the atomic chain, with the thermal vibrations of atoms taken into account:

$$
v_T(\boldsymbol{\rho}) = \frac{1}{d}\int_{-\infty}^{+\infty}\int V_T(\boldsymbol{r}-\boldsymbol{R})\left(\frac{3}{2\pi}\right)^{3/2} R_T^{-3}\exp\left(-\frac{3R^2}{2R_T^2}\right)d^3\boldsymbol{R}dz\ ,
$$

$$
\begin{aligned}
v_T(\boldsymbol{\rho}) = \frac{e}{2m^2\varepsilon}\left(\frac{\mathfrak{m}_T}{m}+\frac{\mathfrak{e}_T}{\varepsilon+m}\right)&((\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{\rho})2g'(\rho^2)(\boldsymbol{\rho}\hat{\boldsymbol{p}})+(\hat{\boldsymbol{p}}\boldsymbol{\rho})2g'(\rho^2)(\boldsymbol{\rho}\cdot\boldsymbol{\sigma}\times\hat{\boldsymbol{p}})\\
&-(\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{a})g(\rho^2)(\boldsymbol{a}\hat{\boldsymbol{p}})-(\boldsymbol{a}\hat{\boldsymbol{p}})g(\rho^2)(\boldsymbol{\sigma}\times\hat{\boldsymbol{p}}\cdot\boldsymbol{a}))\ ,
\end{aligned} \tag{7}
$$

where R_T^2 is root-mean-square amplitude of an atom thermal vibrations. Function $g(\rho^2) = -\frac{1}{\rho}\frac{d\Phi(\rho)}{d\rho}$ is expressed through the ordinary T-even axis potential

$$
\Phi(\rho) = \frac{Ze}{d}\left(E_1\left(\frac{3\rho^2}{2R_T^2+2R_a^2}\right)-E_1\left(\frac{3\rho^2}{2R_T^2}\right)\right)\ , \tag{8}
$$

obtained by the averaging of the screened Coulomb potential $\phi(r)$ in the manner above. $E_1(x) = \int\limits_1^{\infty} t^{-1}e^{-xt}dt$ is exponential integral.

Thus, the particle interaction with the axis consists of T-odd interaction $v_T(\rho)$ and T-even one $v(\rho) = e\Phi(\rho)$.

T-odd amplitude per unit length of an axis can be calculated in the distored waves approximation [14]:

$$
F_T(0) = -\frac{\varepsilon}{2\pi}\frac{1}{d}\int\limits_{-d/2}^{d/2}dz\int d^2\boldsymbol{\rho}\Psi^{*(-)}(\boldsymbol{r})v_T(\boldsymbol{\rho})\Psi^{(+)}(\boldsymbol{r}). \tag{9}
$$

Plane wave distortion by the Coulomb axis potential can be easy taken into account in terms of the eikonal approximation [19] using Schrodinger equation (5):

$$
\Psi^{(+)}(\boldsymbol{r}) = exp\left(\mathrm{i}\boldsymbol{pr}-\mathrm{i}\frac{\varepsilon}{p}\delta(\rho^2,(\boldsymbol{n}_\perp\boldsymbol{\rho}))\right),\quad \Psi^{(-)}(\boldsymbol{r}) = exp\left(\mathrm{i}\boldsymbol{pr}+\mathrm{i}\frac{\varepsilon}{p}\delta(\rho^2,-(\boldsymbol{n}_\perp\boldsymbol{\rho}))\right), \tag{10}
$$

where $\delta(\rho^2,(\boldsymbol{n}_\perp\boldsymbol{\rho})) = \int\limits_{-\infty}^{(\boldsymbol{n}_\perp\boldsymbol{\rho})} v(|\ \boldsymbol{\rho}-\boldsymbol{n}_\perp(\boldsymbol{n}_\perp\boldsymbol{\rho})+\boldsymbol{n}_\perp t\ |)dt$, $\boldsymbol{n}_\perp$ is perpendicular to the axis component of a unit vector in the particle momentum direction. Note, that δ depends from $\boldsymbol{\rho}$ only through combinations ρ^2 and $(\boldsymbol{n}_\perp\boldsymbol{\rho})$, that is corollary of the cylindrical symmetry of the system considered. Apparently, the same T-odd effect is to exist for neutrons also. In this case plane wave distorting by cylindrical potential originates from the neutron magnetic moment interaction with the charged nuclei or from the strong neutron interaction with nuclei. Substituting wave functions (10) and T-odd potential (7) into the equation (9) we obtain:

$$
\begin{aligned}
F_T(0) &= \frac{Ze^2\varepsilon}{2\pi m^2 d}\left(\frac{\mathfrak{m}_T}{m}+\frac{\mathfrak{e}_T}{\varepsilon+m}\right)(\boldsymbol{na})(\boldsymbol{\sigma}\cdot\boldsymbol{a}\times\boldsymbol{n})\\
&\times\frac{1}{n_\perp^2}\int e^{-\mathrm{i}\frac{\varepsilon}{p}(\delta(X,Y)+\delta(X,-Y))}\Big\{\left(2YX\frac{\partial\delta(X,Y)}{\partial X}+Y^2\frac{\partial\delta(X,Y)}{\partial Y}\right)2g'(X)\\
&+\left(2Y\frac{\partial\delta(X,Y)}{\partial X}+n_\perp^2\frac{\partial\delta(X,Y)}{\partial Y}\right)g(X)\Big\}d^2\boldsymbol{\rho}\ ,
\end{aligned} \tag{11}
$$

where $X = \rho^2$ and $Y = (\boldsymbol{\rho} \cdot \boldsymbol{n}_\perp)$.

To simplify the calculations $g(\rho^2)$ and potential $v(\rho)$ have been approximated by the simple functions:

$$g(\rho^2) = g(0)e^{-\alpha\rho^2}, \quad v(\rho) = v(0)e^{-\alpha\rho^2},$$

where $g(0) = \dfrac{Ze}{d}\dfrac{3R_a^2}{(R_T^2 + R_a^2)R_T^2}$, and $v(0) = \dfrac{Ze^2}{d}\ln\left(\dfrac{R_T^2 + R_a^2}{R_T^2}\right)$. For the tungsten the best fitted parameter α, at $R_T = 0.087$ Å and $R_a = 0.27$ Å, appears to be equal $\alpha = 51.4$ Å^{-2}. Distance between atoms is $d = 3.165$ Å for an axis (100). T-odd spin rotation angle [8] ϕ_T (per unit particle path length) in tungsten target and T-odd cross section by an axis σ_T (per one axis atom) for protons with the energy 100 GeV are shown in fig. 3. The quantities involved are proportional to the real and imaginary parts of C_1 from the expression (1) accordingly :

$$\phi_T = \frac{2\pi\wp \mathrm{Re} C_1}{p} n_\perp d, \qquad \sigma_T = \frac{2\pi \mathrm{Im} C_1}{p} n_\perp d \ .$$

From the (11) it follows, that at large energies, when $\frac{\varepsilon}{p} \approx 1$, contribution determined by $\mathfrak{m}_T$ to the T-odd amplitude is proportional to the energy of a particle. So, T-odd cross section and angle of spin rotation, appropriate to this contribution, do not depend on energy. Let's remind, that, according to the eq.(2), T-violating charge $\mathfrak{e}_T$ and magnetic moment $\mathfrak{m}_T$ are measured in terms of e and $e/2m$ accordingly. Results shown in fig.3 are calculated for $\mathfrak{m}_T + \mathfrak{e}_T\left(1 + \frac{\varepsilon}{m}\right)^{-1} = 1$. For this case the T-odd cross section is about 10^{-4} mbarn. Accuracy of cross section measurements reaches 10^{-6} [7] by now, hence, it is possible to obtain restriction 10^{-2} on $\mathfrak{m}_T$. Measurements should be made for angles $\sim 10^{-2} - 10^{-3}$. In doing so frequency of the T-odd scattering amplitude oscillation is not high, in addition, spin depolarisation (relative decreasing of absolute polarisation magnitude on unit particle path length) $\eta = 4\wp\frac{Z^2e^4}{m^2}\left(\mu - \frac{\varepsilon}{\varepsilon+m}\right)^2\frac{R_a - R_T}{d n_\perp}$. is low. The depolarisation arising from scattering by atomic chains is calculated in the manner described in [20] and comes to 6 % in one meter target for the angle $n_\perp = 10^{-2}$.

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FIGURES

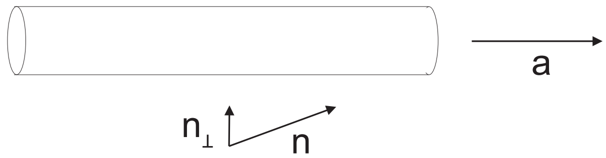


FIG. 1. Particle scattering by the atomic axis.

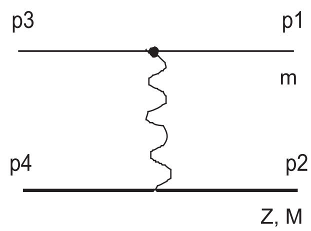
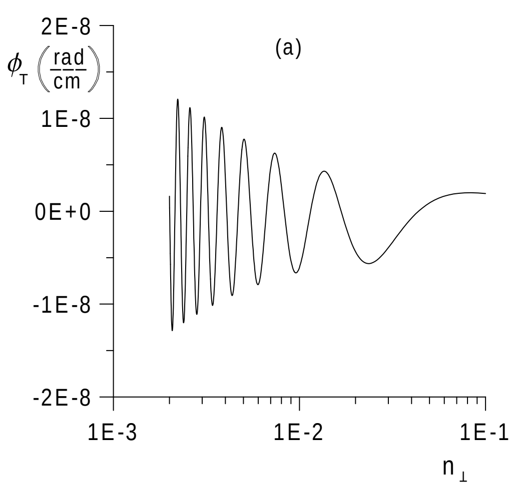


FIG. 2. The diagram of one photon exchange. The T-violating P-even vertex is designated by a black circle.



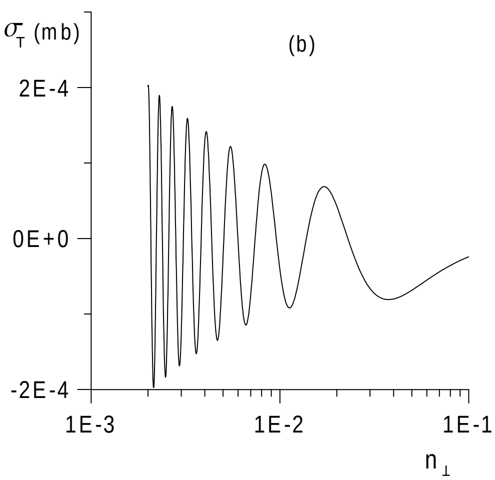


FIG. 3. (a) Angle of T-odd spin rotation of protons with the energy 100 GeV in tungsten target of 19.4 g/cm^3 density, (b) T-odd cross section (related to one axis atom) of a proton by (100) tungsten crystal axis . $n_\perp$ is a perpendicular to the axis component of the unit Vector in a direction of proton motion.